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Particle Boundedness in $\Gamma^i_{jk;l} = 0$, $g_{ij;k} = 0$ Field Theory

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Abstract

We have studied the problem of particle boundedness in the field theory based on the equations $\Gamma^i_{j;kl} = 0$ and $g_{ij;k} = 0$. We have found no evidence of boundedness. However, this negative result may be the result of using too simplified a solution to the integrability equation.

1. *Introduction*

In a previous series of papers (Muraskin, 1970, 1971a, b, 1972; Muraskin & Clark, 1970; Muraskin & Ring 1971), we introduced the field equations $\Gamma^i_{jk;l} = 0$, $g_{ij;k} = 0$ and discussed some simple properties of these equations.

We have found that a minimum in g_{00} can be made to appear at an arbitrary origin. The usefulness of this effect, so far as a model of a particle, would certainly depend on whether a bound could be found for the particle. That is, as we proceed down the x axis, for example, g_{00} gets larger. It is necessary that eventually g_{00} must stop increasing. It is not clear how far we would have to go in order to see this effect, since there is no scale appearing in the field equation.

2. *Reduction of Errors*

In order to make long computer runs, we have improved our program for reducing errors. We shall first describe what we have done in this regard.

We have calculated the field components using a particular grid size. Then, we have repeated the calculation using $\frac{1}{2}$-grid, $\frac{1}{4}$-grid, $\frac{1}{8}$-grid and $\frac{1}{16}$-grid. We then compared the field at the first point on the original grid. We have found that the following rules are rather closely obeyed. We denote by Δ^i_{jk} the difference between a field component using the original grid size with the same field component using the $\frac{1}{2}$-grid at this first point. This difference is then halved when we compare the results for the $\frac{1}{2}$-grid with the results for the $\frac{1}{4}$-grid. This rule is still obeyed as we continue to half the grid. According to the above observation, we get a considerably improved Γ^i_{jk} at the first point appearing in the original grid calculation if we take

$$\Gamma^i_{jk} - \Delta^i_{jk}(1 + \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \cdots) = \Gamma^i_{jk} - 2\Delta^i_{jk}.$$

Here, the infinite series of corrections has been summed. This rule, we note, holds quite well for all components Γ^i_{jk}. To see how good the corrected gamma is [where $(\Gamma^i_{jk})_{\text{corrected}} = \Gamma^i_{jk} - 2\Delta^i_{jk}$], we compared the corrected gamma with the corrected gamma when the calculation is repeated with the grid halved. The results of this gave numbers ranging from 10^{-11} to 10^{-14}, depending in which components we were looking at. This is about a 10^4 improvement over the difference as calculated with uncorrected gammas. We then chose $(\Gamma^i_{jk})_{\text{corrected}} = \Gamma^i_{jk} - 2{\cdot}0018\Delta^i_{jk}$. When we compared this expression with a similar expression for the $\frac{1}{2}$-grid, we got magnitudes similar to the above, but with changes of sign for all components. This tells us that our factor of 2 is fairly accurate. In long runs, we have checked periodically that this factor-of-2 observation remains valid. We have found that it does. We have noted also that there is some slight build-up of the 10^{-11} to 10^{-14} number as we proceed down the x axis (for example, for Γ^1_{11} we have $0{\cdot}19 \times 10^{-13}$ goes to $0{\cdot}21 \times 10^{-13}$ for a 300 point stretch at $x = 0{\cdot}60$). In order to reduce the error further, we have made a subtraction every 300 points. For example, suppose for one component this number starts at $0{\cdot}19 \times 10^{-13}$ and after 300 points it is $0{\cdot}21 \times 10^{-13}$. Then, we subtracted from this component

$$\left(\frac{0{\cdot}21 + 0{\cdot}19}{2}\right) \times 10^{-13} \times 300 \times 2$$

after 300 points. The 2-factor was inserted as an estimate of the effect of an infinite series of corrections.

We have made a test on whether the integrability equations are still valid. We compared the field by starting from $x = 1{\cdot}11$ and then preceding first down the y axis and then down the z axis, and vice versa, for fourteen points in each direction. We found that the field was the same independent of the path to, on the order of, thirteen decimal places.

We recall that what we are looking for is a turnabout in some component of the field [by turnabout, we mean that the field component stops increasing (decreasing) and starts to decrease (increase)]. We have already seen a turnabout in our previous computer analysis of the field equations. Particle A in our previous work can be seen to exhibit a turnabout by studying the numbers on any of the six planes tabulated. By purposefully introducing greater errors by making the grid larger, we found that the turnabout still occurred—although it was slightly displaced in space. This is an argument that the turnabout effect is not overly sensitive to error.

3. *Results*

For initial parameters, we chose the following

$$e^1{}_1 = 0{\cdot}24 \qquad e^1{}_2 = 0{\cdot}25 \qquad e^1{}_3 = 0{\cdot}35 \qquad e^1{}_0 = 3{\cdot}1$$
$$e^2{}_1 = -0{\cdot}02 \qquad e^2{}_2 = -0{\cdot}03 \qquad e^2{}_3 = -0{\cdot}023 \qquad e^2{}_0 = 0{\cdot}082$$
$$e^3{}_1 = -0{\cdot}03 \qquad e^3{}_2 = -0{\cdot}024 \qquad e^3{}_3 = -0{\cdot}015 \qquad e^3{}_0 = 0{\cdot}092$$
$$e^0{}_1 = 0{\cdot}12246 \qquad e^0{}_2 = 0{\cdot}13995 \qquad e^0{}_3 = 0{\cdot}15490 \qquad e^0{}_0 = 2{\cdot}0$$

$$\Gamma^0_{00} = 4{\cdot}0$$
$$A = 0{\cdot}6$$
$$B = D = -0{\cdot}7$$
$$C = -1{\cdot}1$$
$$E = 1{\cdot}0$$

where (no summation over repeated indices a, b, c, 1, 2, 3)

$$A = \Gamma^a_{aa}$$
$$B = \Gamma^a_{bc} \qquad a, b, c \text{ all different}$$
$$C = \Gamma^a_{ab} \qquad a \neq b$$
$$D = \Gamma^a_{ba} \qquad a \neq b$$
$$E = \Gamma^a_{bb} \qquad a \neq b$$

$e^0{}_1$, $e^0{}_2$ and $e^0{}_3$ are rounded off above. This set of parameters leads to a minimum for g_{00} at the origin point.

We have run 11,100 points down the x axis. This took a total close to 2 days. We ran at the grid 0·001. Where the field components all do not agree to five decimal places with the results for the $\frac{1}{2}$-grid, the computer automatically reduces the grid size by half till agreement is reached. However, this feature of the program never came into explicit use, since over the 11,100 points we always got agreement to five decimal places. The field components varied by several times their original value over this range. For example, Γ^2_{00} started out at −87·34 and ended up at 639·56 (these figures are rounded off). This is the largest total change of any component.

The trends that we observed were as follows. None of the Γ^i_{jk} showed any turnabout in this range. All the components of Γ^i_{jk} that started off getting larger, continued to do so. All the Γ^i_{jk} that started off getting smaller, continued to do so. Furthermore, the rate of increase (decrease) continued to grow throughout. Thus, there is no sign of any turnabout for any of the components $\Gamma^i_{jk} \cdot g_{00}$ exhibited similar behavior. Its initial value was −5·63. At 11,100 points, its value was 6·55.

Thus, we conclude that after 11,100 points there is absolutely no sign of any boundedness for our particle-like behavior. Thus, we have the following possibilities.

(1) Although no sign of turnabout has shown up, this may not mean that it will not appear eventually if we continue down the x axis. At the expense of increasing errors, we increased our grid by a factor of 50 to see the general trend ahead. We proceeded from $x = 1{\cdot}11$ to $x = 1{\cdot}71$ at this grid. We can have confidence in our figures to the order of one decimal place. We have found that the trends described above continue.

(2) Our initial data at the origin point may be inadequate. We have never been able to obtain the general solution of the integrability equations. It may be that we lose important information by taking simplified solutions to the 96 algebraic non-linear equations. An example of a solution of the integrability equations that leads to singularities developing is the case of all Γ^i_{jk} being equal.

Thus, we conclude that a bad choice of parameters at the initial point can lead to singularities developing. That is, global behavior depends on the solution of the integrability equations that we employ.

(3) It may be that all solutions of $\Gamma^i_{jk;l} = 0$ lead to singularities developing away from the origin. On the other hand, in view of the reasonable local properties we have found up to now, and the 'aesthetic' motivation, it may be that the equations should be capable of leading to reasonable global properties as well.

4. *Outlook*

The basic problem, we feel, is to find general solutions of the integrability equations. This, unfortunately, is mathematically rather complicated.

There is an interesting possibility so far as motivating the choice of initial parameters. We could require that all invariants containing Γ^i_{jk} be zero at the origin. We have previously pointed out that these invariants are constant. The problem of whether these conditions can be met will be discussed at a later time.

5. *Conclusions*

We cannot say we have fully explored the consequences of the field equations. However, our meager attempts thus far do not give evidence that the equations give rise to a reasonable model of a particle. However, this negative result may be a consequence of using too simple a solution to the integrability equations.

The difficulty of obtaining a bound may also be reflected by the fact that a reasonable particle is 'denser' than the region around it by many orders of magnitude. For example, the density of a proton is $\sim 10^{12}$ gm/cm^3. Thus, the difference between numbers describing the particle with numbers associated with vacuum may be enormous.

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